Ioffe Institute
6th Int. Symp. "Nanostructures: Physics and Technology"
St Petersburg, Russia, June 22–26, 1998.



# Density of states of tunnel-coupled 2D electron layers in strong magnetic fields

*O. E. Raichev*
Institute of Semiconductor Physics, NAS Ukraine, Pr. Nauki 45, Kiev, Ukraine

**Abstract.** The density of states for a pair of disordered tunnel-coupled 2D electron layers in the quantum Hall regime is calculated. The interplay between the scattering-induced broadening of the Landau levels and tunnel-induced hybridization of the electron states is demonstrated. The conditions for the tunneling gap, when a single Landau level splits in two levels, are expressed through the parameters of the system. It is shown that even a symmetrical double-layer system in a strong magnetic field acquires a substantial dipole moment due to interlayer asymmetry of the disorder.

## Introduction

Electronic properties of double-layer two-dimensional (2D) electron systems formed in double quantum wells [1] (DQW's) or in wide wells [2] have attracted much attention in past years. In particular, it is found that the tunnel coupling of electron states in these systems is important for interpretation of the quantum Hall effect data [2-4], since it opens the tunneling gap separating the symmetric and antisymmetric (with respect to the layer index) states. Despite the intensive studies, the influence of the tunnel coupling on the properties of double-layer systems is not fully disclosed. Theoretical studies of the equilibrium properties and in-plane transport coefficients of the DQW's with a pronounced tunnel coupling have been done for the case of weak magnetic fields only.

In this paper we study the density of states of the electron gas in tunnel-coupled DQW's in a strong magnetic field taking into account the Landau level broadening due to elastic scattering of the electrons. Apart from the density of states, we calculate another important function, the interlayer distribution density, which describes the difference between the concentrations in the wells (in the same way as the density of states describes the total concentration). Both these functions can be used for determination of all the equilibrium quantities (for example, specific heat, magnetic susceptibility and capacitance) characterizing the double-layer system.

## 1 Basic outline of the calculations

The calculations described below are done by two methods. The first is the self-consistent Born approximation (SCBA), see [5]. The SCBA gives a reasonable description of the density of states of 2D systems for any kind of elastic disorder, but it is not an exact method. For this reason, we also apply an exact method, based upon the path integral formalism, in the "adiabatic" approximation, when a characteristic length of the inhomogeneities exceeds the magnetic length.

Consider a double-layer system in the magnetic field $H$ applied perpendicular to the layers. In the basis of left $(l-)$ and right $(r-)$ layer orbitals, the Hamiltonian of the system is

$$\frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial \mathbf{x}}+\frac{e}{c}\mathbf{A}\right)^2+\varepsilon_s+\hat{h}+\hat{V}(\mathbf{x}), \tag{1}$$

where $\mathbf{A}=[\mathbf{H}\times\mathbf{x}]/2$ is the vector-potential of the magnetic field, $\varepsilon_s$ is the Zeeman energy ($s$ is the spin index), $\hat{h}$ is the potential energy matrix of the system, and $\hat{V}(\mathbf{x})$ is the matrix of the disorder potential (we consider the elastic disorder, which, besides, has no influence on the electron spin). The potential energy matrix is given as $\hat{h}=(\Delta/2)\hat{\sigma}_z+T\hat{\sigma}_x$, where $\hat{\sigma}_i$ are the Pauli matrices, $\Delta$ is the splitting energy in the absence of tunnel coupling, and $T$ is the tunneling matrix element determining the minimum splitting energy between the symmetric and antisymmetric states. The matrix of the disorder potential is $\hat{V}(\mathbf{x})=\hat{P}_l V_l(\mathbf{x})+\hat{P}_r V_r(\mathbf{x})$, where $\hat{P}_l=(1+\hat{\sigma}_z)/2$ and $\hat{P}_r=(1-\hat{\sigma}_z)/2$ are the projection matrices. The density of electron states $N(\varepsilon)$ and the interlayer distribution density $N_z(\varepsilon)$ are expressed through the retarded matrix Green function $\hat{G}^R_\varepsilon(\mathbf{x},\mathbf{x}',s)$ of the Hamiltonian (1):

$$\begin{pmatrix} N(\varepsilon)\\ N_z(\varepsilon)\end{pmatrix}=-\frac{1}{\pi}\sum_s Im\ Tr\begin{pmatrix}1\\ \hat{\sigma}_z\end{pmatrix}\left\langle \hat{G}^R_\varepsilon(\mathbf{x},\mathbf{x},s)\right\rangle, \tag{2}$$

where $Tr$ means the matrix trace, and the statistical averaging $\langle\ldots\rangle$ is carried over all possible configurations of the random potentials $V_l(\mathbf{x})$ and $V_r(\mathbf{x})$.

**A**. Within the SCBA, the averaged Green function of the Hamiltonian (1) is

$$\hat{G}_\varepsilon(n,s)=\left[\varepsilon-\varepsilon_n-\varepsilon_s-\hat{h}-\hat{\Sigma}_\varepsilon(n,s)\right]^{-1}. \tag{3}$$

We use the Landau level representation, $n$ is the Landau level number, $\varepsilon_n=\hbar\omega_c(n+1/2)$, $\omega_c$ is the cyclotron frequency, and $\hat{\Sigma}_\varepsilon(n,s)$ is the self-energy matrix. The last is given by the following equation

$$\hat{\Sigma}_\varepsilon(n,s)=\sum_{n',j,j'}\Phi_{jj'}(n,n')\hat{P}_j\hat{G}_\varepsilon(n',s)\hat{P}_{j'}, \tag{4}$$

where $j=l,r$ is the layer index. The scattering probabilities $\Phi_{jj'}(n,n')$ are expressed through the Fourier transforms of the random potential correlators $W_{jj'}(|\mathbf{x}-\mathbf{x}'|)=\langle V_j(\mathbf{x})V_{j'}(\mathbf{x}')\rangle$.

After a substitution of the Green function (3) into Eq.(4), we obtain a set of equations needed for determination of the self-energy. We assume the case of very strong magnetic fields, when the cyclotron energy is large in comparison with all other energies. This allows us to consider each Landau level separately ($n'=n$ in Eq.(4)). In this approximation, Eqs.(3) and (4) give a set of nonlinear algebraic equations, which are to be solved numerically, giving, thereafter, $N(\varepsilon)$ and $N_z(\varepsilon)$.

**B**. The path-integral representation [6] of the retarded Green function is

$$\hat{G}^R_\varepsilon(\mathbf{x},\mathbf{x}',s)=-\frac{i}{\hbar N}\int_0^\infty dt\exp\left(\frac{i}{\hbar}(\varepsilon-\hat{h})t\right)\int_{\mathbf{x}=\mathbf{x}_0}^{\mathbf{x}'=\mathbf{x}_t}\mathcal{D}[\mathbf{x}_\tau]$$
$$\times\exp\left\{\frac{i}{\hbar}\int_0^t d\tau[L_s(\mathbf{x}_\tau,\dot{\mathbf{x}}_\tau)-\hat{V}(\mathbf{x}_\tau)]\right\}, \tag{5}$$

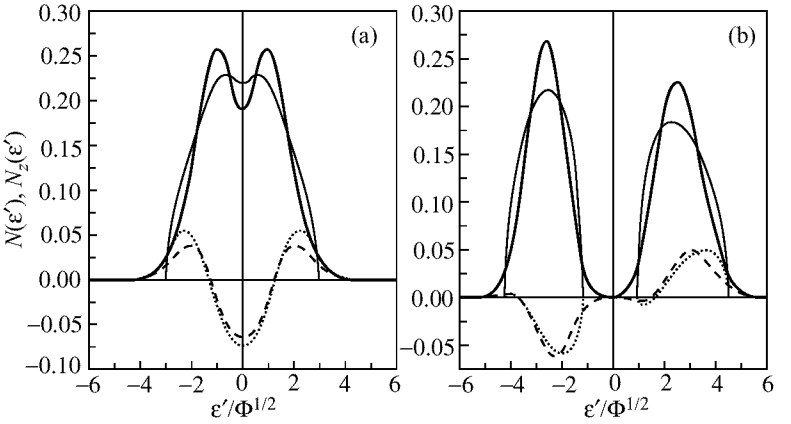


**Fig 1.** (a) Density of states $N(\varepsilon')$ (solid) and interlayer distribution density $N_z(\varepsilon')$ (dash) in units of $1/\pi a_H^2\sqrt{\Phi}$ at $\mu = 0.6$, $\gamma = 0.3$, $\Delta = 0$, and $T = \sqrt{\Phi}$. (b) The same for $\Delta = \sqrt{\Phi}$ and $T = 2.5\sqrt{\Phi}$. The SCBA results are shown by thin lines.

where $N$ is the normalization factor, and $L_s(\mathbf{x}, \dot{\mathbf{x}}) = m\dot{\mathbf{x}}^2/2 + e\mathbf{H}[\mathbf{x}\times\dot{\mathbf{x}}]/2c - \varepsilon_s$ is the free-particle Lagrangian. The direct statistical averaging of the matrix exponential expression in Eq.(5) is impossible, in contrast to scalar exponentials [6]. For this reason, some rigid approximations are done from the very beginning. We assume that the characteristic length of the inhomogeneities considerably exceeds the electronic wavelength which, in a strong magnetic field, is equal to the magnetic length $a_H$. Then, we can replace $\hat{V}(\mathbf{x}_\tau)$ by $\hat{V}$, and the path integral gives

$$N(\varepsilon') = \frac{1}{2\pi a_H^2}\left\langle \delta(\varepsilon' - V_0 - V_T) + \delta(\varepsilon' - V_0 + V_T)\right\rangle, \tag{6}$$

$$N_z(\varepsilon') = \frac{1}{2\pi a_H^2}\left\langle \frac{\Delta/2 + V_z}{V_T}[\delta(\varepsilon' - V_0 - V_T) - \delta(\varepsilon' - V_0 + V_T)]\right\rangle, \tag{7}$$

where $\varepsilon' = \varepsilon - \varepsilon_n - \varepsilon_s$ is the energy counted from the center of each spin-split Landau level, $V_0 = (V_l + V_r)/2$, $V_z = (V_l - V_r)/2$, and $V_T = \sqrt{T^2 + (V_z + \Delta/2)^2}$. The averaging in Eqs.(6) and (7) is to be done with use of a two-dimensional Gaussian distribution function [6], which is expressed through the correlators $W_{jj'}(0)$.

## 2 Results and discussion

Below we introduce the values $\Phi = [\Phi_{ll}(n,n) + \Phi_{rr}(n,n)]/2$, $\mu = [\Phi_{ll}(n,n) - \Phi_{rr}(n,n)]/[\Phi_{ll}(n,n) + \Phi_{rr}(n,n)]$ and $\gamma = \Phi_{lr}(n,n)/\Phi$. The first characterizes averaged (over the layer index) scattering within $n$-th Landau level, while the second and the third characterize the asymmetry and interlayer correlation of the disorder potential, respectively. In the case of smooth disorder, the scattering probabilities $\Phi_{jj'}(n,n')$ are given as $\Phi_{jj'}(n,n') = \delta_{nn'}W_{jj'}(0)$, and one can express $\Phi$, $\mu$ and $\gamma$ directly through $W_{ll}(0)$, $W_{rr}(0)$ and $W_{lr}(0)$. This allows to make a comparison between the results of the SCBA and path-integral method.

Fig.1 (a) shows energy dependence of $N(\varepsilon')$ and $N_z(\varepsilon')$ calculated by both methods in symmetrical conditions, $\Delta = 0$. At a finite tunnel coupling, the Landau level formed by two coupled states (the "coupled Landau level") begins to split. As the coupling

increases (larger $T$), a complete splitting occurs and the tunneling gap opens up, see Fig.1 (b). The opening of the gap leads to an additional quantized Hall plateau when the Fermi level stays in the gap. In the SCBA, the density of states has sharp edges, and one can write a strict condition for the tunneling gap. At $\Delta = 0$ the gap exists when

$$T^2 > \Phi(1+\gamma-\lambda)^2/(1-\lambda), \quad \lambda = |\mu|\sqrt{(1-\sqrt{1-\mu^2})/(1+\sqrt{1-\mu^2})}. \qquad (8)$$

At $\Delta \neq 0$, a similar analytical expression can be found for symmetrical disorder ($\mu = 0$): $T^2 > [\Phi - (\Delta/4)^2](1+\gamma)^2$. Of course, these expressions should be applied in a qualitative sense only, because the edges of the density of states cannot be as sharp as the SCBA predicts. However, they give a correct information about the influence of the disorder parameters on the tunnel splitting of the Landau levels (this is justified by the path-integral calculations). In particular, Eq.(8) shows that the interlayer correlation ($\gamma$) suppresses the gap, while the interlayer asymmetry of the disorder potential ($\mu$) favors the gap.

The interlayer distribution density $N_z(\varepsilon')$ at $\Delta = 0$ is a symmetric function with respect to the center of the coupled Landau level. It is not equal to zero because of the disorder asymmetry which means that the double-layer electron has a dipole moment even at $\Delta = 0$. At $\Delta \neq 0$, $N_z(\varepsilon')$ exists both for $\mu = 0$ and $\mu \neq 0$. Nevertheless, the dipole moment, which is proportional to the energy integral of $N_z(\varepsilon')$, goes to zero when the coupled Landau level is fully occupied.

In conclusion, we have calculated the density of states and interlayer distribution density for a pair of tunnel-coupled 2D layers in a strong magnetic field. The Landau level broadening is described with use of the self-consistent Born approximation and the path-integral approach for smooth Gaussian disorder. A possible interlayer asymmetry of the disorder potential is taken into account. The conditions for the tunneling gap are expressed through the parameters of the system.

## References

[1] J. P. Eisenstein, *Superlattices and Microstructures* **12**, 107 (1992); N. K. Patel *et. al.*, *Semicond. Sci. Technol.* **11**, 703 (1996).
[2] Y. W. Suen *et. al.*, *Phys. Rev. Lett.* **68**, 1379 (1992).
[3] S. Q. Murphy *et. al.*, *Phys. Rev. Lett.* **72**, 728 (1994).
[4] T. S. Lay *et. al.*, *Phys. Rev. B* **50** 17725 (1994).
[5] T. Ando *et. al. Rev. Mod. Phys.* **54**, 437 (1982).
[6] V. L. Bonch-Bruevich *et. al.*, *Electron Theory of Disordered Semiconductors* (Nauka, Moscow, 1981).